UDC 538.9

BRIGHT SOLITONS IN OPTICAL MEDIA WITH HIGH ORDER EFFECTS

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Abstract. *In this article bright soliton solutions of the generalized nonlinear Schrödinger equation are found with the help of an effective potential, which is found too. The equation accounts second and fourth order dispersion and also third and fifth order nonlinearity of the media. For the obtained solutions their existence and stability regions are determined. The stability regions are verified and confirmed by solving the equation numerically.*

Key words: *Nonlinear medium, dispersion, slowly varying envelope, effective potential, phase portrait, soliton.*

UO‘K 538.9

YUQORI TARTIBLI EFFEKTLARGA EGA OPTIK MUHITLARDA YORQIN SOLITONLAR

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Annotatsiya. *Ushbu maqolada umumlashgan nochiziqli Schrödinger tenglamasining yorug‘ soliton yechimlari aniqlangan. Buning uchun topilgan effektiv potensialdan foydalanilgan. Tenglamada muhitlarning ikkinchi va to‘rtinchi tartibli dispersiyasi bilan birga uchinchi va beshinchi tartibli nochiziqliligi hisobga olingan. Aniqlangan yechimlar uchun ularning mavjudlik va turg‘unlik sohalari aniqlangan. Turg‘unlik sohalari tenglamani sonli yechish orqali tekshirilgan va tasdiqlangan.*

Kalit so‘zlar: *Nochiziqli muhit, dispersiya, sekin o‘zgaruvchi amplituda, effektiv potensial, fazaviy portret, soliton.*

УДК 538.9

СВЕТЛЫЕ СОЛИТОНЫ В ОПТИЧЕСКИХ СРЕДАХ С ЭФФЕКТАМИ ВЫСШЕГО ПОРЯДКА

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Аннотация. *В данной статье определены светлые солитонные решения обобщённого нелинейного уравнения Шредингера с помощью найденного эффективного потенциала. Это уравнение учитывает дисперсии второго и четвертого порядко, а также третьего и пятого порядков нелинейности среды. Для полученных решений определены их области существования и устойчивости. Области устойчивости проверены и подтверждены решением уравнения численно.*

Ключевые слова: *нелинейная среда, дисперсия, медленно меняющаяся амплитуда, эффективный потенциал, фазовый портрет, солитон.*

Introduction

It is known that the dependence of the refractive index on the frequency leads to dispersion. Dispersion of the group velocity leads to the spreading of pulses. Cubic nonlinearity can balance spreading due to quadratic dispersion, resulting to the formation of stable optical pulses, solitons [1]. For frequencies close to zero of quadratic dispersion, higher-order dispersion terms become important. Recently, it has been shown theoretically and experimentally that a balance between fourth-order dispersion and cubic nonlinearity also gives stable optical pulses [2]. The propagation of pulses in media with higher dispersion is described by the following equation [3, 4]

$$i\psi_t - \frac{\beta_2}{2}\psi_{xx} + \frac{\beta_4}{24}\psi_{xxxx} + \gamma_1|\psi|^2\psi + \gamma_2|\psi|^4\psi = 0, \tag{1}$$

where $\psi(x, t)$ is the slowly varying envelope of the electric field, β_2 and β_4 are second and fourth order dispersion coefficients, γ_1 and γ_2 are coefficients of cubic and quintic nonlinearity, x and t are spatial and temporal coordinates, respectively and $i^2 = -1$. The properties of the media are determined by dispersion and nonlinear coefficients.

Materials and Methods

Because of stationary solutions are shape preserving solutions we look for stationary solutions in the form of real profile function multiplied by $\exp(-i\omega t)$. After substituton we acquire ordinary differential equation instead of partial differential equation. It should be stressed that, for this, owing to field in the infinity is zero, integration constant should be zero too. Solving last equation gives us desired solution. Existence regions are domain of definition of obtained solution. To determine stability regions firstly we use linear stability analysis and found spectral problem. For solving the spectral problem numerically used Fourier collocation method. And we check the stability regions with the help of split-step method.

Results

Stationary solutions. We look for stationary solutions or so-called bright soliton solutions of equation (1) in the following form

$$\psi(x,t) = u(x)\exp(-i\omega t), \tag{2}$$

where $u(x)$ is a real function and ω is frequency. By substituting equation (2) in equation (1), the ordinary differential equation is obtained from the partial differential equation

$$\omega u - \frac{\beta_2}{2}u_{xx} + \frac{\beta_4}{24}u_{xxxx} + \gamma_1 u^3 + \gamma_2 u^5 = 0. \tag{3}$$

This equation is fourth order and to find its general solution is not simple. However, there are some ways to find one of the solutions which the equation is satisfied. If we multiply the equation to u_x and simplify, the following conservation law obtains

$$\omega u^2 - \frac{\beta_2}{2}u_x^2 + \frac{\beta_4}{24}\left(2u_{xxx}u_x - u_{xx}^2\right) + \frac{\gamma_1}{2}u^4 + \frac{\gamma_2}{3}u^6 = C_0, \tag{4}$$

where C_0 is zero, because field in the infinity is zero. Taken equation is third order, but it is not convenient to solve yet. Fortunately, if we accept the following assumption that the squared u_x is in the form of m-th order polynomial $P_m(u)$, i.e.,

$$u_x^2 = F(u) \equiv P_m(u), \tag{5}$$

the equation converts to solvable and very useful one. The equations (4) and (5) are reminiscent of the equation of motion of a classical particle in a potential. Therefore, we adopt an effective potential in which an effective particle moves

$$U(x) = -\frac{1}{2}F(u). \tag{6}$$

With the help of the assumption all derivatives in equation (4) can be expressed by polynomial

$$u_{xx} = \frac{1}{2}F_u, \quad u_{xxx}u_x = \frac{1}{2}F F_{uu}. \tag{7}$$

Now, we rewrite equation (4) taking into account equation (7)

$$\omega u^2 - \frac{\beta_2}{2}F + \frac{\beta_4}{24}\left(F F_{uu} - \frac{1}{4}F_u^2\right) + \frac{\gamma_1}{2}u^4 + \frac{\gamma_2}{3}u^6 = 0. \tag{8}$$

From equation (8) we find the order of unknown polynomial. It is obvious that the order of polynomial in the bracket in equation (8) is (2*m* - 2) and it should be balanced by 6. Finally, *m* is 4 and the polynomial has the following form

$$u_x^2 = F(u) \equiv P_m(u) = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4, \tag{9}$$

where a_0, a_1, a_2, a_3 and a_4 are unknown coefficients that have to be determined by the parameters of nonlinear media. After substituting equation (9) in equation (8), we equalize all coefficients to zero and find that a_0, a_1 and a_3 are zero and

$$a_2 = \frac{6}{5\beta_4}\left(\beta_2 + \gamma_1\sqrt{\frac{-\beta_4}{\gamma_2}}\right), \quad a_4 = -\sqrt{\frac{-\gamma_2}{\beta_4}}, \quad \omega = \omega_-,$$

$$a_2^* = \frac{6}{5}\left(\frac{\beta_2}{\beta_4} + \frac{\gamma_1}{\gamma_2}\sqrt{\frac{-\gamma_2}{\beta_4}}\right), \quad a_4^* = \sqrt{\frac{-\gamma_2}{\beta_4}}, \quad \omega^* = \omega_+, \tag{10}$$

$$\omega_\pm = \frac{3}{50\beta_4\gamma_2}\left[9\beta_2^2\gamma_2 + \beta_4\gamma_1\left(\gamma_1 \pm 8\beta_2\sqrt{\frac{-\gamma_2}{\beta_4}}\right)\right].$$

An effective potential function corresponding to a bright soliton should satisfy the conditions: it has an extremum and its second derivative is less than zero at the origin of coordinate; it has at least a positive zero [9]. The potential and its phase portrait in phase plane (u, u_x) is depicted in figure (1). In phase plane O_1 and B_1 points are saddle point and center, respectively, and A_1 point defines the amplitude of a bright soliton. And also blue and green lines are phase trajectories that separated by a red line so-called separatrix.

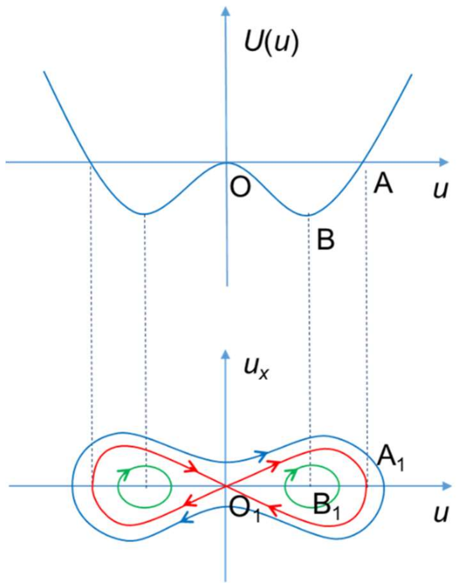

Figure 1: An effective potential vs. *u* (upper) and a suitable phase portrait (lower). Points O and B correspond to unstable and stable positions of an effective particle and point A defines the amplitude of a bright soliton.

As a consequence, we choose the first one of equation (10) and the effective potential is

$$U(u) = au^4 + bu^2, \quad F(u) = -2U(u),$$
$$a = \frac{1}{2}\sqrt{\frac{-\gamma_2}{\beta_4}}, \quad b = \frac{-3}{5\beta_4}\left(\beta_2 + \gamma_1\sqrt{\frac{-\beta_4}{\gamma_2}}\right), \tag{11}$$

We solve the ordinary differential equation given by equation (9) taking into account equation (11). Consequently, we obtain bright soliton solutions of equation (3) in explicit form

$$u(x) = A\mathrm{sech}(kx),$$
$$A = \sqrt{\frac{-b}{a}} = k\left(-\frac{\beta_4}{\gamma_2}\right)^{1/4}, \quad \omega = \frac{\beta_2}{2!}k^2 - \frac{\beta_4}{4!}k^4, \tag{12}$$
$$k^2 = \frac{6}{5\beta_4}\left[\beta_2 + \gamma_1\sqrt{\frac{-\beta_4}{\gamma_2}}\right].$$

Existence and stability of the pulse. From the analysis of an effective potential or, equivalently, equation (12), it is obvious that the signs of coefficients of fourth order dispersion and quintic nonlinearity are opposite as well as k^2 is positive. Using the results of the analysis we determine the regions of existence of bright solitons in plane (β_4, γ_2).

Stability of a solition is important because only stable solitons can be observed experimentally. In order to determine the linearly stable and unstable regions of solitons we use the linear-stability analysis and the Fourier collocation method [10]. In figure (2) the existence and stable regions are presented in four cases.

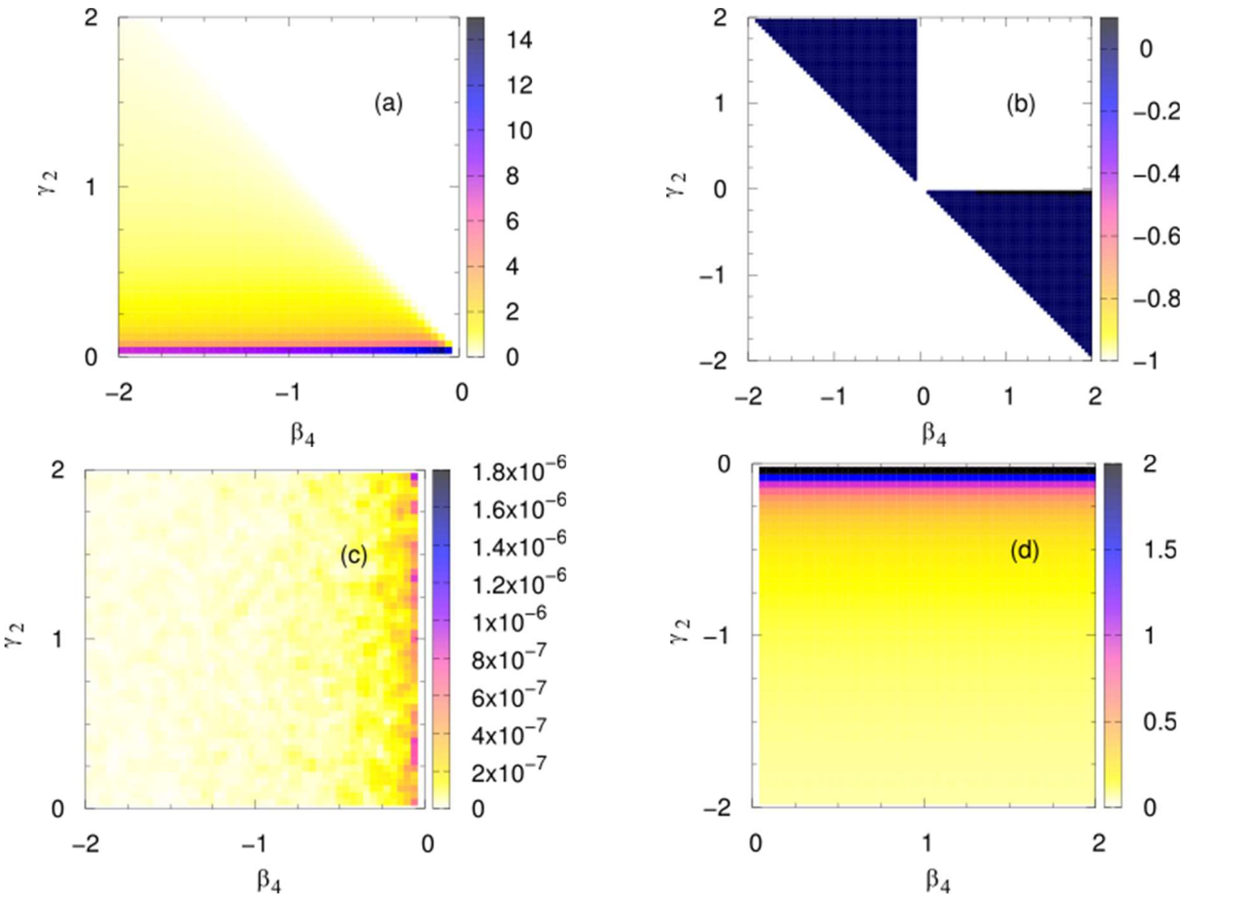

Figure 2: Existence and stability regions of bright solitons. Here, (a) (β_2, γ_1) = (-1, -1); (b) (β_2, γ_1) = (-1, 1); (c) (β_2, γ_1) = (-1, 0); (d) (β_2, γ_1) = (0, 1). Solitons do not exist only in the case of (b) in white regions, in all other cases solitons exist. Solitons are stable in the following cases: (a) in white regions; (b) and (c) only in coloured regions. In all other regions solitons are unstable.

To demonstrate the propagation of stable and unstable solitons we use split-step method to solve the ordinary differential equation numerically [6]. In figures (3) and (4) the dynamics of stable and unstable bright solitons and time evolutions of their parameters are presented.

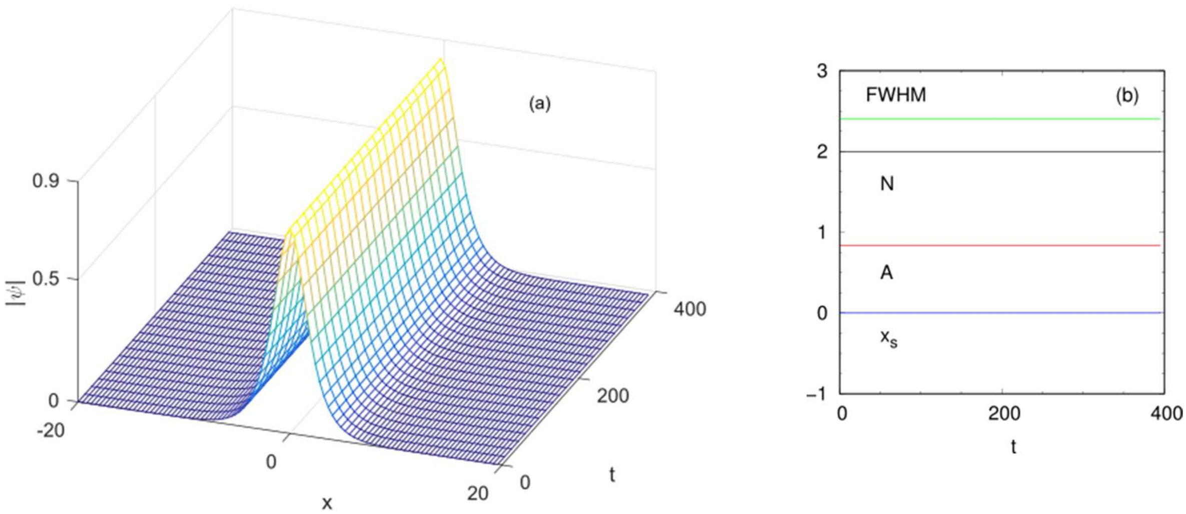

Figure 3: (a) Dynamics of a stable bright soliton and (b) time evolution of its parameters. Labels FWHM, N, A and x_s stand for full width at half maximum, norm, amplitude and soliton center, respectively. Here, $(\beta_2, \gamma_1, \beta_4, \gamma_2) = (-1, 1, 1, -0.5)$.

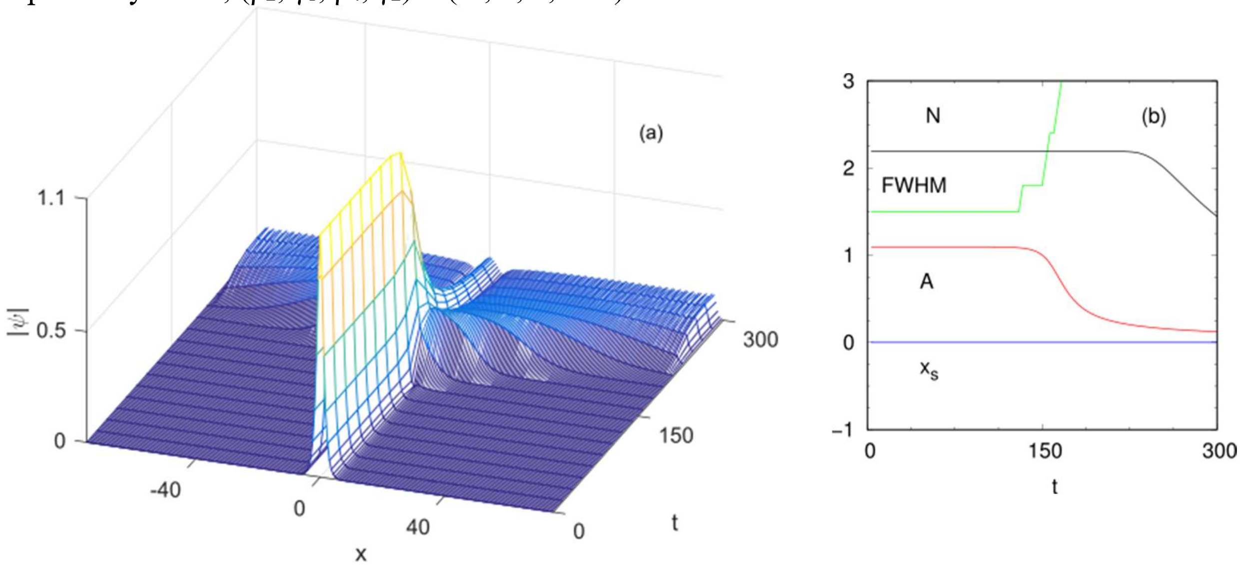

Figure 4: (a) Dynamics of an unstable bright soliton and (b) time evolution of its parameters. Labels N, FWHM, A and x_s stand for norm, full width at half maximum, amplitude and soliton center, respectively. Here, $(\beta_2, \gamma_1, \beta_4, \gamma_2) = (0, 1, 1, -1)$.

Discussion

Obtained solution is known but its properties are not sufficiently investigated. In other works stationary solution is mathematically derived in different ways and its physical properties are ignored. For example, in Refs. [6], [7] and [8] this is exactly the case. In our studies apart from stationary solutions ussues of existence and stability are also addressed. Furthermore, stability regions are checked by numerically solving the pulse propagation equation.

Conclusion

In this paper, exact solutions for bright solitons in optical media with high order dispersion and nonlinearity are found. These solitons exist only for opposite signs of the parameters of fourth-order dispersion and fifth-order nonlinearity. The regions of existence and stability of solitons are obtained.

The results can be useful for experiments on generating of stable optical pulses in media with higher-order effects.

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